Beech Hyde Primary School and Nursery

Maths Calculation Policy

Introduction

This policy outlines the school's agreed progression through written strategies for addition subtraction, multiplication and division. Children work through the progression so that they can understand, use, apply and explain a compact method of calculation by the end of Year 6, although it is expected that children throughout the school will be able to explain and apply the current mental and written methods they are using. As children will move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However we would expect the majority of each class to be working at age-appropriate levels as set out in the National Curriculum.

The policy also includes examples and diagrams, showing how we teach calculations, as consistency in layout and presentation is important to support learning and progression. The policy also includes the equipment and resources that will be used to support children's understanding of the strategy.

The importance of mental maths

While this policy focuses on written calculations in maths, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

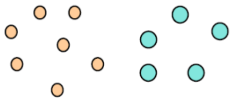
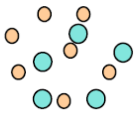
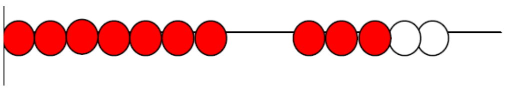
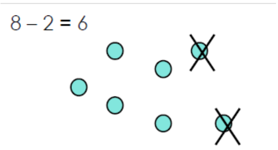
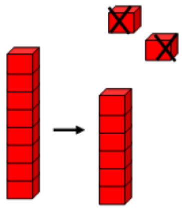
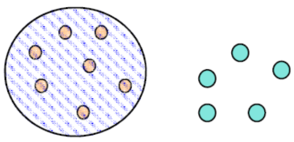
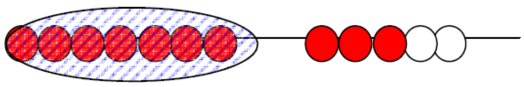
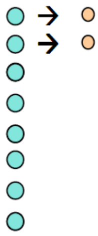
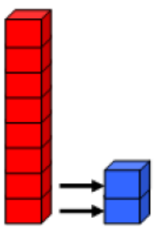
To add and subtract successfully, children should be able to:

- Recall all addition pairs to 9 + 9 and number bonds to 10
- Recognise addition and subtraction as inverse operations
- Add mentally a series of one digit numbers (e.g. 5 + 8 + 4)
- Add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 — 70)
- Partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)

To multiply and divide successfully, children should be able to:

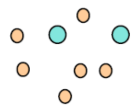
- Recall all multiplication facts to 12 x 12
- Calculate products such as 50 x 7, 50 x 70, 700 x 5 or 700 x 50 using the related fact 7 x 5 and their knowledge of place value, including patterns of multiplying and dividing by the powers of 10
- Add 2 or more single digit numbers mentally
- Partition 2 and 3 digit numbers into multiple of 100, 10 and 1 in different ways
- Add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value
- Recognise multiplication and division as inverse operations
- Know subtraction facts to 20 and use this knowledge to subtract multiples of 10 (e.g. 120—80, 320—90)
- Use tables knowledge to estimate how many times one number divided into another (e.g. how many 6s there are in 47 or how many 23s there are in 92)

Progression in Addition and Subtraction

ADDITION	SUBTRACTION
Combining two sets (aggregation model) Putting together – two or more amounts or numbers are put together to make a total. Using a wide range of objects; including objects of mixed sets and Dienes units.  Count one, then the other. Combine the sets and count again. Starting at 1.  Counting along the bead bar, count out the 2 sets and then draw them together. 	**Taking away (separation model)** Where one quantity is taken away from another to calculate what is left.  Multilink towers- to physically take away objects. 
Combining two sets (augmentation model) Increasing – when one amount is made bigger Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.  Counters- 7 then count on 8, 9, 10, 11, 12  Number tracks- start on 7 and then count on 5 more jumps.  Bead strings- make a set of 7 and a set of 5. Then count on from 7.	**Finding the difference (comparison model)** Two quantities are compared to find the difference. 8 – 2 = 6  Cuisenaire Rods 8  3+5 Multilink towers 

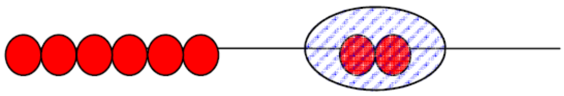
1 set within another (part-whole model)

The quantity in the whole set and one part are known, and may be used to find out how many are in the unknown part.



Bead strings

8 – 2 = 6

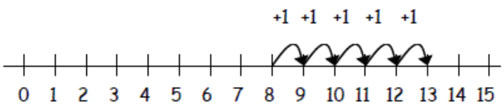


Bridging through 10 U+U, TU+U

This method teaches children to become more efficient.

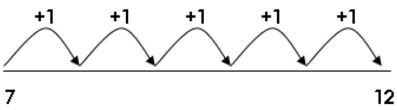
Labelled number line

8 + 5 = 13



Blank number line

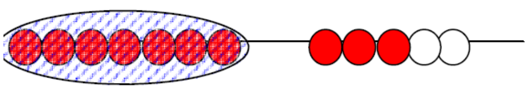
7 + 5 = 12



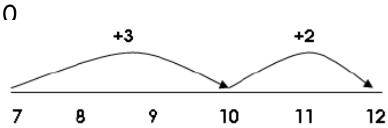
7 + 5 is broken down into 7 + 3 + 2 using questions such as 'How many more to the next multiple of 10?' and then 'If we use 3 of the 5 to get to 10, how many more do we need to add on?'

This encourages children to use and apply their knowledge of number bonds to 10.

Bead strings are used to illustrate addition including bridging through a tens boundary by counting on 3 and then 2 more.



Steps in addition can be recorded on a number track and number line. The steps often bridge through a multiple of 10

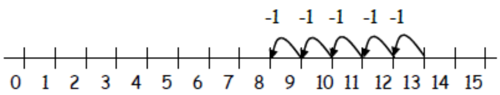


Bridging through 10 TU-U

This method teaches children to become more efficient.

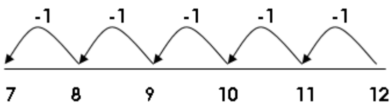
Labelled number line

13 – 5 = 8



Blank number line

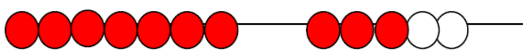
12-5 = 7



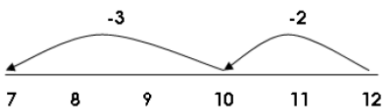
12 – 5 is broken down into 12 – 2 – 3 using questions such as 'How many more to the last multiple of 10?' and then 'If we use 2 of the 5 to get to 10, how many more do we need to subtract?'

This encourages children to use and apply their knowledge of number bonds to 10.

Bead strings are used to illustrate subtraction including bridging through a tens boundary by counting back 2 and then 3 less.



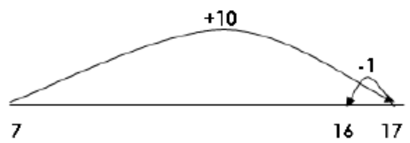
Steps in subtraction can be recorded on a number track and number line. The steps often bridge through a multiple of 10.



Compensation method TU+U, TU+TU

Children are also taught this method when they are adding numbers close to a multiple of 10, such as 9, 11, 19, 21, etc.

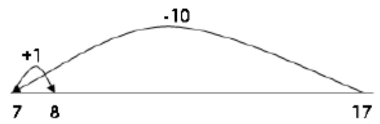
For example, 7 + 9 would be treated as 7+10 and then adjusted by subtracting 1.



Compensation method

Children are also taught this method when they are subtracting numbers close to a multiple of 10, such as 9, 11, 19, 21, etc.

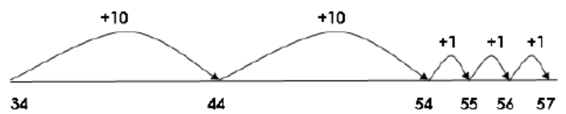
17 – 9 is treated as 17 – 10 and then adjusted by adding 1.



Working with larger numbers- partitioning TU+TU

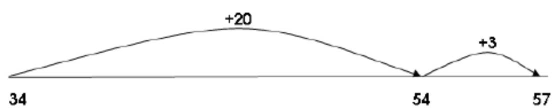
Children will continue to use empty number lines, starting with the larger number and counting on. They may use compensation where appropriate.

34 + 23 = 57

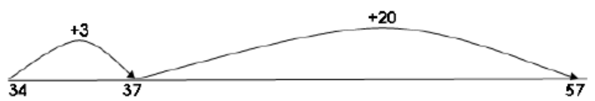


When comfortable, children will be encouraged to combine the tens into a single jump and then the units into another jump (or 2 jumps if crossing a tens boundary).

34 + 23 = 57



To prepare children for working with Dienes equipment, we then ensure children always start working with the units.

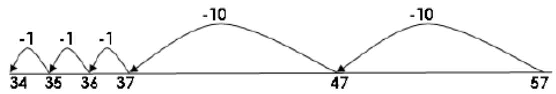


All of these strategies continue to be supported through using bead strings

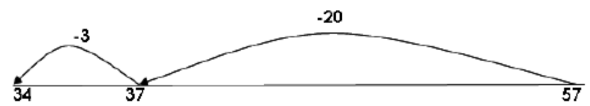
Working with larger numbers- partitioning TU-TU

Children will continue to use empty number lines, starting with the larger number and counting on. They may use compensation where appropriate.

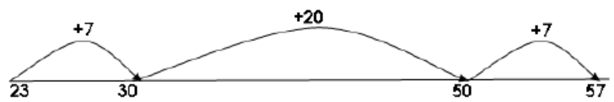
57 – 23 = 34



57 – 23 = 34



The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 23 to 57 in steps totaling 34.

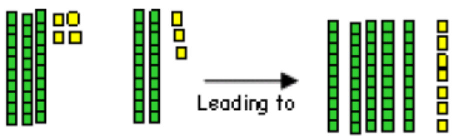


All of these strategies continue to be supported through bead strings.

Aggregation model of addition

Children combine 2 sets, units with units and tens with tens, starting with 1 (or 10). We need to ensure that children always start working with the units.

34 + 23 = 57

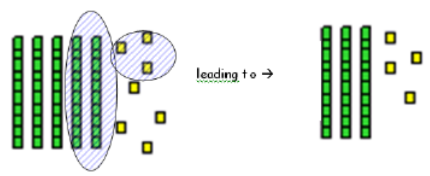


When children are confident with this model of addition, we move them back to the augmentation model.

Separation model of subtraction

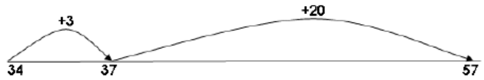
Children combine 2 sets, units with units and tens with tens, starting with 1 (or 10). We need to ensure that children always start working with the units.

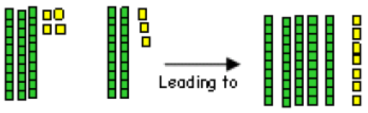
57 – 23 = 34



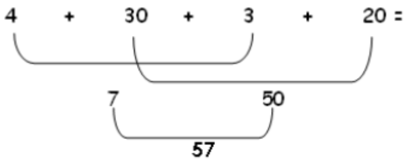
Augmentation model of addition

Encourage children to begin to count out from the first set of units and tens, avoiding the need to start at 1 (or 10).



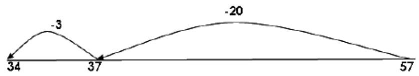


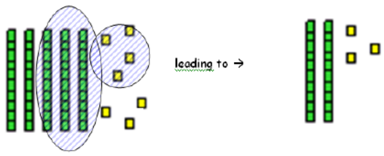
At this stage, children can begin to use an informal method to support record and explain their methods.



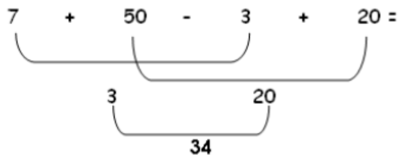
Comparison and part-whole models of subtraction

Finding the difference, or counting on. Find the difference between 57 and 34.





At this stage, children can begin to use an informal methods.

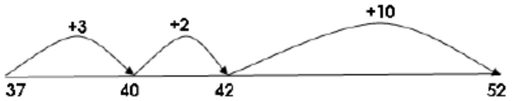


Bridging through 10 with larger numbers.

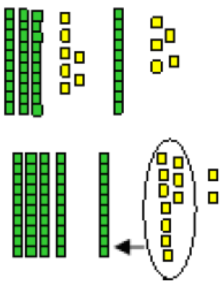
Once secure in partitioning for addition, children begin to explore exchanging. What happens if the units are greater than 10? Introduce the term 'exchange'. Using the Dienes equipment, children exchange ten units for a tens rod, which is equivalent to crossing the tens boundary on the number line.

We encourage children to adopt the same approach of getting to the next multiple of 10 first, then adding the rest of the units, and finally adding the tens. This allows children to use their knowledge of number bonds.

37 + 15 = 52



37 + 15 = 52

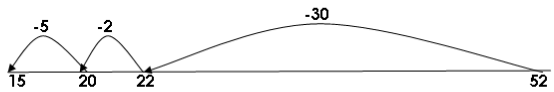


Bridging through 10 with larger numbers

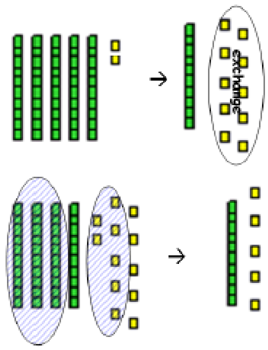
Once secure in partitioning for subtraction, children return to exchanging. What happens if there are not enough units? Introduce the term 'exchange'. Using the Dienes equipment, children exchange ten units for a tens rod, which is equivalent to crossing the tens boundary on the number line.

We encourage children to adopt the same approach of getting to the previous multiple of 10 first, then subtracting the rest of the units, and finally subtracting the tens. This allows children to apply their knowledge of number bonds.

52 – 37 = 15



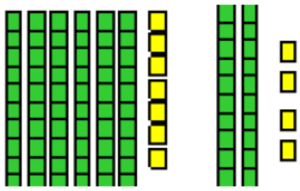
52 – 37 = 15

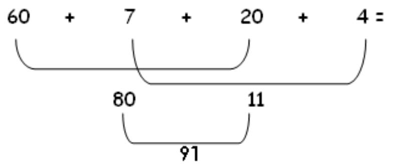


Expanded vertical method

Children are then introduced to the expanded vertical method to ensure that they make the link between using Dienes equipment, partitioning recording and the expanded vertical method.

67 + 24 = 91





This layout shows the addition of the tens to the tens and the units to the units separately. To find the partial calculations children add the units first, and the total of the partial calculations can be found by recombining the sums of the tens and units.

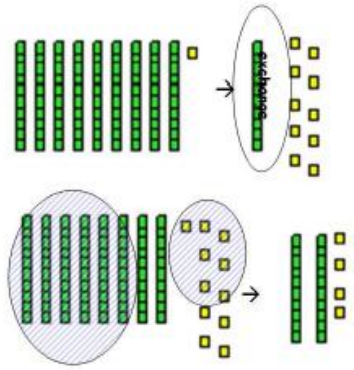
The addition of the tens in the calculation 67 + 24 is described in the words 'sixty plus twenty equals eighty', stressing the link to the related fact 'six plus two equals eight'.

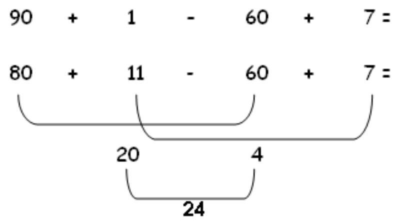
	67	60 + 7
+	24	20 + 4
	11	7 + 4
	80	60 + 20
	91	

Expanded vertical method

Children are then introduced to the expanded vertical method to ensure that they make the link between using Dienes equipment, partitioning recording and the expanded vertical method.

91 – 67 = 24





Partitioning the numbers into tens and units and method, where units are placed under units and tens under tens

	80		11
	~~90~~	+	~~1~~
-	20	+	4
	60	+	7

Compact method

The expanded method leads children to the more compact method so that they understand its structure and efficiency.

In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

	67
+	24
	91
	1

Compact method (decomposition)

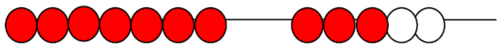
The expanded method leads children to the more compact method so that they understand its structure and efficiency.

Children are able to use the compact method, crossing tens, hundreds boundaries and dealing with zeros.

	8 10
	91
-	67
	24

Addition of decimals

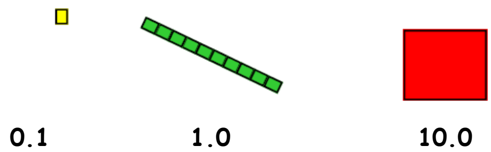
We return to the models that the children are familiar with.
Ensure that the children are confident in counting decimals.



Counting: lots of practicing backwards and forwards, etc.

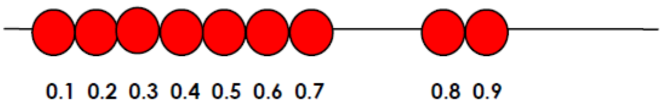
Bead strings: each bead representing 0.1

Dienes equipment: each different block of colour is 1.0



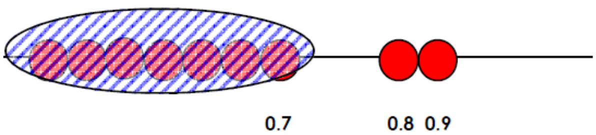
Aggregation model of addition: counting both sets– starting at zero.

0.7 + 0.2 = 0.9



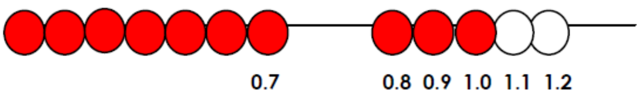
Augmentation model of addition: starting from the first set total, count on to the end of the second set.

0.7 + 0.2 = 0.9

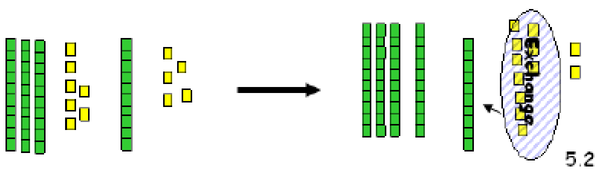


Bridging through 1.0: encouraging connections with number bonds.

0.7 + 0.5 = 1.2



Partitioning
3.7 + 1.5 = 5.2

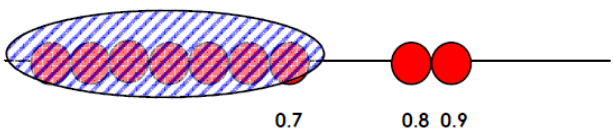


Subtraction of decimals

Separation model of subtraction

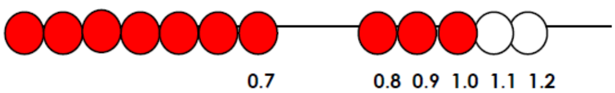
0.9 - 0.7 = 0.2

Comparison model of subtraction

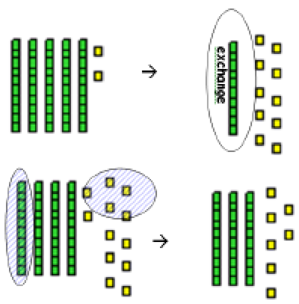


Bridging through 1.0: encouraging connections

1.2 - 0.5 = 0.7



Partitioning
5.2 - 1.5 = 3.7



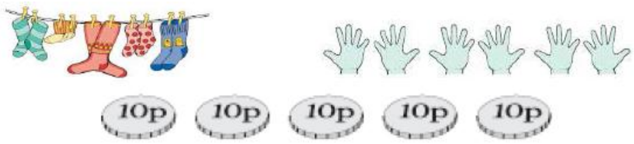
Gradation of difficulty- addition	Gradation of difficulty- subtraction
1. No exchange 2. Extra digit in the answer 3. Exchanging units to tens 4. Exchanging tens to hundreds 5. Exchanging units to tens and tens to hundreds 6. More than two numbers in calculation 7. As 6 but with different number of digits 8. Decimals up to 2 decimal places (same number of decimal places) 9. Add two or more decimals with a range of decimal places.	1. No exchange 2. Fewer digits in the answer 3. Exchanging tens for units 4. Exchanging hundreds for tens 5. Exchanging hundreds to tens and tens to units 6. More than two numbers in calculation 7. As 6 but with different number of digits 8. Decimals up to 2 decimal places (same number of decimal places) 9. Subtract two or more decimals with a range of decimal places.

Progression in Multiplication and Division

<table>
<tr><th>MULTIPLICATION</th><th>DIVISION</th></tr>
<tr><td>

Early experiences

Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.



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Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.

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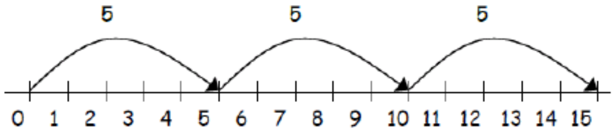
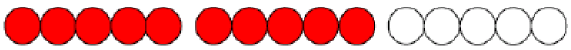
Repeated addition (repeated aggregation)

3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3

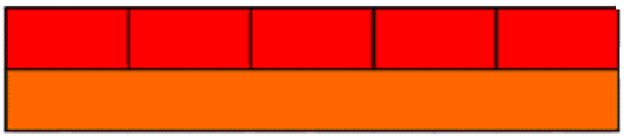
Children learn that repeated addition can be shown on a number line.



Children learn that repeated addition can be shown on a bead string.



Children also learn to partition totals into equal trains using Cuisenaire Rods.

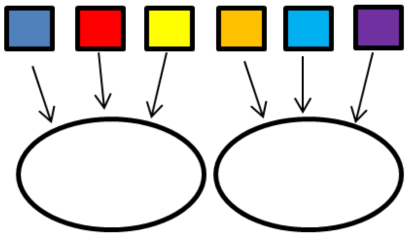


5 x 3 = 15

</td><td>

Sharing equally

6 sweets get shared between 2 people. How many sweets do they each get?



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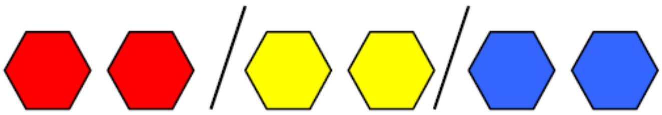
Grouping or repeated subtraction

There are 6 sweets. How many people can have 2 sweets each?



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</table>

Scaling

This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles.
This can be written as:

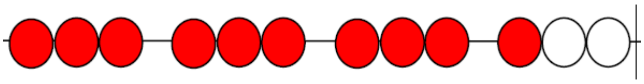
1 + 1 + 1 = 3 → scaled up by 3 → 5 + 5 + 5 = 15

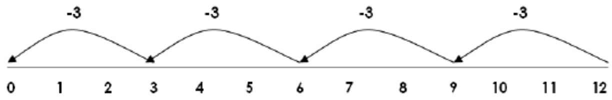
For example, find a ribbon that is 4 times as long as the blue ribbon.

We should also be aware that if we multiply by a number less than 1, this would correspond to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to 3 X 0.5 = 1.5.

Repeated subtraction using a bead string or number line

12÷3 = 4





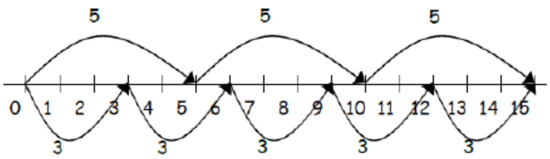
The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as 'how many 3s make 12?'

Cuisenaire Rods also help children to interpret division calculations.

Commutativity

Children learn that 3 x 5 has the same total as 5 x 3. This can also be shown on the number line.

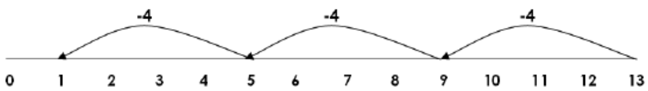
3 x 5 = 15
5 x 3 = 15



Grouping involving remainders

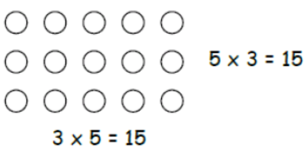
Children move onto calculations involving remainders.

13 ÷ 4 = 3 r1



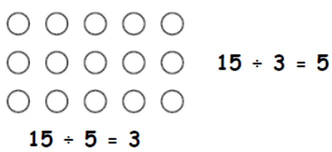
Arrays

Children learn to model a multiplication calculation using an array. This model supports their understanding of commutativity and the development of the grid in a written method.



Arrays

Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method.



Inverse operation

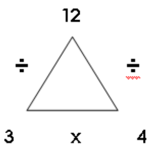
Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts.

3 x 4 = 12

4 x 3 = 12

12 ÷ 3 = 4

12 ÷ 4 = 3



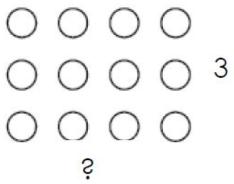
Children use symbols to represent unknown numbers and complete equations using inverse operations.

They use this strategy to calculate the missing numbers in calculations.

□ x 5 = 20   3 x Δ = 18   ○ x □ = 32

This can also be supported using arrays:

e.g. 3 X ? = 12



Inverse operation

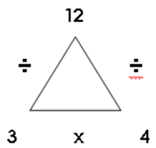
Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts.

12 ÷ 3 = 4

12 ÷ 4 = 3

3 x 4 = 12

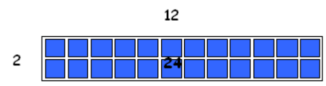
4 x 3 = 12

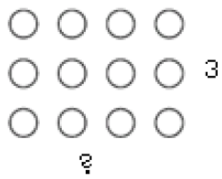


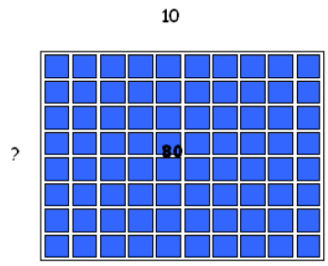
Children use symbols to represent for unknown numbers to complete equations using inverse operations.

They use this strategy to calculate the missing numbers in calculations.

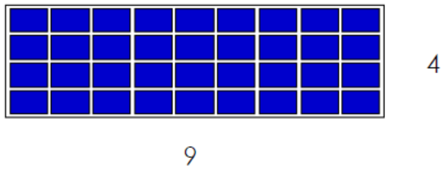
24 ÷ 2 = □   15 ÷ Δ = 3   ○ ÷ 10 = 8



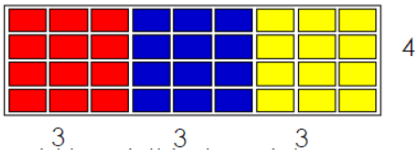




Partitioning
Arrays are also useful to help children visualise how to partition larger numbers into more useful arrays.

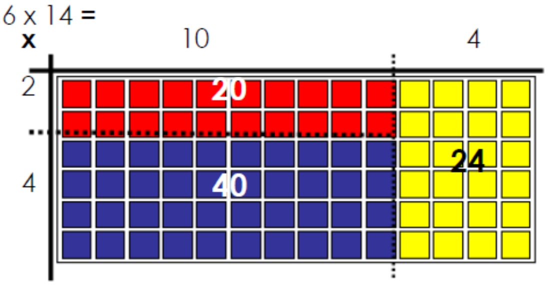


9 x 4 = 36



Children could break this down into more manageable arrays
9 x 4 = (3 x 4) + (3 x 4) + (3 x 4)
= 12 + 12 + 12
= 36

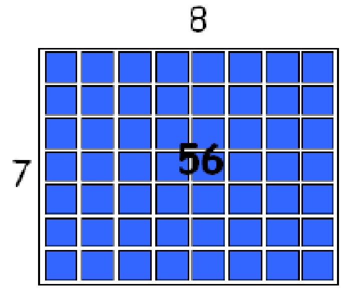
6 x 14 =



6 x 14 =
= (2 x 10) + (4 x 10) + (4 x 6)
= 20 + 40 + 24
= 84

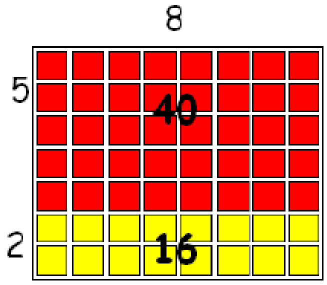
Partitioning
Arrays are also useful to help children visualise how to partition larger numbers into more useful arrays.

56 ÷ 8 = 7



Children could break this down into more manageable arrays, as well as using their understanding of the inverse relationship between division and multiplication.

56 ÷ 8 =



56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8)
= 5 + 2
= 7

Arrays leading into the grid method
Children continue to use arrays and partitioning where appropriate, to prepare them for the grid method of multiplication.

Arrays can be represented as 'grids' in a shorthand version.

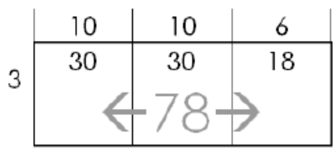
38 x 5 =

x	10	10	10	8
5	50	50	50	40

38 x 5 =
= (10 x 5) + (10 x 5) + (10 x 5) + (8 x 5)
= 50 + 50 + 50 + 40
= 190

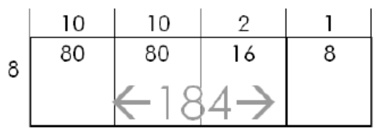
Arrays leading into chunking
Children continue to use arrays and partitioning where appropriate, to prepare them for the 'chunking' method of division. Arrays are represented as 'grids' as a shorthand version.

78 ÷ 3 =



78 ÷ 3 =
= (30 ÷ 3) + (30 ÷ 3) + (18 ÷ 3)
= 26

184 ÷ 8 =



So 184 ÷ 8 = 23

Grid method

This written strategy is introduced for the multiplication of TU x U to begin with. It uses clear column addition methods to calculate the total.

23 x 8 =

Children will approximate first.

23 x 8 is approximately 25 x 8 = 200.

x	20	3
8	160	24

```
    160
+    24
    184
```

The vertical method- 'chunking'

78÷3

```
      78
-     30     (10 x 3)
      48
-     30     (10 x 3)
      18
-     18     (6 x 3)
       0
```

So 78 ÷ 3 = 10 + 10 + 6 =26

184 ÷ 8 =

```
     184
-     80     (10 x 8)
     104
-     80     (10 x 8)
      24
-     16     (2 x 8)
       8
-      8     (1 x 8)
       0
```

Dealing with remainders

Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly.

e.g.

I have 62p. How many 8p sweets can I buy?

Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed?

HTU x U

346 x 9

Approximate first:

346 x 9 is approximately 350 x 10 = 3500

x	300	40	6
9	2700	360	54

```
    2700
+    360
+     54

    3114
     1 1
```

TU ÷ U -compact methods

Children continue to use chunking methods to solve division calculations using increasingly more efficient methods and larger multiples of the divisor (20x, 30x).

196 ÷ 6 =

```
     196
-    180     (30 x 6)
      16
-     12     (2 x 6)
       4
```

196 ÷ 6 = 32 r 4

Children learn to show quotients as fractions.

196 ÷ 6 = 32 4/6 or 32 2/3

TU x TU

72 x 38

Approximate first:

72 x 38 is approximately 70 x 40 = 2800

x	70	2
30	2100	60
8	560	16

```
    2100
+    560
+     60
+     16
    2736
      1
```

HTU ÷ TU

977÷36 =

```
    977
-   720     (20 x 36)
    257
-   252     (7 x 36)
      5
```

977÷36 = 27r5 or 275/7

U.t x U

4.9 x 3

Approximate first:

4.9 x 3 is approximately 5 x 3 = 15

x	4	0.9
3	12	2.7

```
    12
+  2.7
  14.7
```

U.t ÷ U

87.5 ÷ 7 =

```
    87.5
-   70.0     (10 x 7)
    17.5
-   14.0     (2 x 7)
     3.5
-    3.5     (0.5 x 7)
       0
```

87.5 ÷ 7 = 12.5

Children extend their use of the grid method to include:

ThHTU x U e.g. 4346 x 8 =

HTU x TU e.g. 372 x 24

U.t h x U e.g 4.92 x 3

Division with remainders (written as a decimal)

Children will learn how to write a division calculation with the remainder written with up to 2 decimal places.

Grid method leading into short multiplication

Children will learn to describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.

```
   38
×   7
  210
   56
  266
```

The recording is reduced further, with carry digits recorded below the line.

```
   38
×   7
  266
   5
```

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded.

Long multiplication

TU x TU 56 × 27 is approximately 60 × 30 = 1800.

56	
× 27	
1000	50 × 20 = 1000
120	6 × 20 = 120
350	50 × 7 = 350
42	6 × 7 = 42
1512	
1	

Which moves on to HTU x TU:

286	
× 29	
4000	200 × 20 = 4000
1600	80 × 20 = 1600
120	6 × 20 = 120
1800	200 × 9 = 1800
720	80 × 9 = 720
54	6 × 9 = 54
8294	
1	

Then moving on to a reduced number of steps, using children's knowledge of place value and number facts:

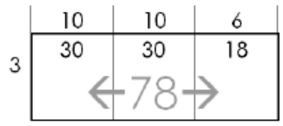
286	
× 29	
5720	286 × 20
2574	286 × 9
8294	
1	

In year 6 pupils will be expected to multiply numbers with up to 4 digits e.g. ThHTU x TU

Long division 'The bus stop method'

This brings children back to using a shorthand version of arrays, combined with skills developed through chunking.

78 ÷ 3 =



becomes

3 | 78

26
3 | 78
- 60
18
- 18
0

196 ÷ 6 =

32 r 4
6 | 196
- 180
16
- 12
4

In year 6 pupils will be expected to divide numbers with up to 4 digits e.g. ThHTU ÷ TU

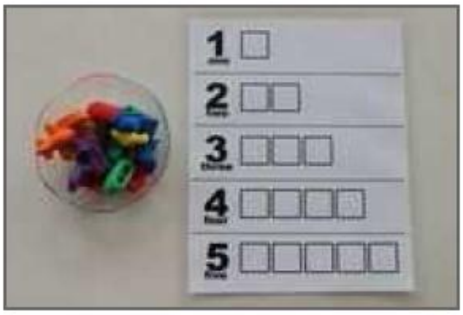
Appendix 1

Counting

The importance of counting can not be overstressed. Children need to be confident in the following areas:

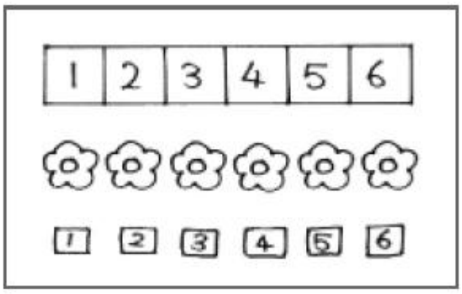
One-to-one correspondence

Children synchronise their counting and pointing, keeping track of their counting as they go, assigning one number name to one object and only counting each object once. Counting static pictures is harder, and children need to devise a system to know which they have counted as they go along.



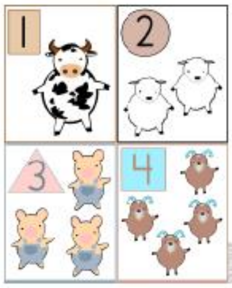
Stable order of counting

To be able to count means knowing that the list of words used must be in a repeatable order. This principle calls for the use of a stable list that is at least as long as the number of items to be counted; if children only know the number names up to 'six', then they obviously are not able to count seven items.



Cardinal aspect of number

This is the idea that when children are counting a set of objects, the last number counted is the number of objects altogether.



Abstract principle of number

This is where children are counting things that cannot be touched or moved, such as sounds, imaginary objects or even the counting words.

Order irrelevance

Children need to understand that the order the objects are counted in doesn't matter and show that when necessary they self-correct or check their counting. It does not really matter whether the counting procedure is carried out from left to right, from right to left or from somewhere else, so long as every item in the collection is counted once and only once.

APPENDIX 2

Partitioning

Children need to feel confident partitioning numbers in different ways.

Greater than/ less than/ equivalent
How many more or less using direct comparison.

= as a sign of equality

Balance scales, using weighted objects, such as Cuisenaire rods, Dienes blocks.

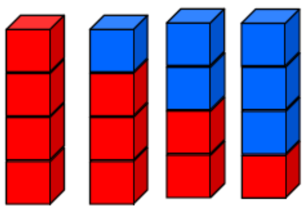
Cuisenaire Roads– make trains of equal length.

Make both sides balance.

Children need to be confident in partitioning numbers in a variety of ways to ensure that they can think flexibly about numbers.

Partitioning can be explored through a wide variety of equipment and materials, all of which can represent a range of numbers and decimals.

Examples of partitioning
All complements to 1, 10, 100 etc.
Partitioning numbers in all possibilities
9
0 + 9
1 + 8
2 + 7
3 + 6
4 + 5
Plus an infinite number of decimal combinations.



Multilink towers– make 4

Bead strings

Cuisenaire Rods– make trains of equal length

Always encourage 'Finding All Possibilities'
(systematic working and how do you know you have them all?)